

## **Alternative limited communication systems: centralization versus interchange of information**

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### **1 Introduction**

This chapter is a contribution to the study of optimal organizational design, a subject of interest to economists for many years but one in which little actual progress has been made. Organizations function by communicating information and coordinating the actions of their members. Both of these activities require the establishment and use of channels for transmitting information between members. The pattern of these communication links is called the organizational structure, or design.

More complex designs allow a fuller sharing of information and a more precise implementation of the desired collective decision. Neglecting the costs of the design and the communication, the more elaborate designs can achieve a higher expected payoff for the organization. Comparing organizational designs requires the computation of the costs of communication itself. It is at this point that economic theory has failed to provide a basis for the analysis, and it is for this reason that the literature on organizational design has not yielded rigorous theoretical results.

Costs of an organizational structure should include all the basic activities in which the organization is involved: collection, storage, retrieval, transmission, and processing of both quantitative and nonquantitative information. The only cost that economists have dealt with, to date, is the cost of transmission. And even here, the metrics in which costs are reckoned are terribly simple. It is in general assumed that each real number transmitted costs the same. The questions typically addressed have been of the form: How many transmissions are needed in order to achieve a particular desired standard of performance? Usually, an efficiency criterion such as Pareto optimality has been the standard.

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This approach to organizational design is quite different from what the early writers had in mind.<sup>1</sup> They wanted to judge different designs against each other in order to measure the benefits from additional resources devoted to the organization's internal network. But without a theory of the costs of alternative design, this program was not feasible.

The primary framework used to date to study the organization design problem on a rigorous basis has been the theory of teams. As set forth in *The Economic Theory of Teams* (Marschak and Radner 1972), the goal of this theory is to be able to compare alternative designs by a two-step method. First, for each given design, we find the optimal mode of functioning for the organization. Second, the cost of operating the communication process is to be subtracted from the optimized objective function of each design, and their net values compared. *The Economic Theory of Teams* did not actually carry out this program. It did set up some useful definitions, and it provided some important results on the first of these steps. But the second step, cost assessment and comparison, was not really attempted, despite a clear view of the problem. Marschak and Radner (1972) are led to the following types of assumptions to validate their analysis. When they compare information structures based on fixed or variable partitions of the team's members into groups, they assume that costs of information depend only upon the average group size. When they compare error in instruction with error in observation, they assume that costs depend upon the ratio of the variance of the error to the variance of the message.

The more recent literature on economic organization has proceeded in two other directions, but has likewise neglected the design problem. Conflicting objectives among the organization's members have been introduced, giving rise to the extensive literature on incentives and the economics of information. And the asymptotic behavior of certain decision rules has been studied as the size of organization grows larger.

In this chapter we return to the original goal in the spirit of Marschak and Radner's Chapter 8. We compare organizational designs that use the *same* resources, thus circumventing the problem of measuring the cost of these resources. We retain the number of communication links in the organization as the measure of the resources needed. In particular, we will assume that the costs of coding and decoding are identical and do not involve increasing returns. This is well in the tradition of team theory. As stressed by Kenneth Arrow (1982): "Team theory assumes a fixed amount of communication in fixed channels. The costs of communication are modelled by scarcity."

We compare two forms of a two-member organization. Each form requires that two pieces of information be sent and received. They differ in that in the first, which we call centralization, one member sends two pieces of information to the other. In the second, which we call interchange, they each send and receive a single piece of data. Thus, under interchange, neither member is as well informed as the receiving agent would be under centralization.

Our results concern the choice between these two communication systems as the team's objective function and its stochastic environment vary. If the costs of malcoordination of the team members' actions are sufficiently high, the member with the more accurate observation on each exogenous variable should send it to the other. This results in centralization if one agent is the better observer of all variables, and in interchange otherwise. *Thus, the superiority of the communication system called centralization requires, in this model, both superiority of information by one player and relatively high costs of malcoordination.*

In Section 2 we describe the basic model to be used. Sections 3 and 4 give analyses of the optimal utilization of interchange and centralization, respectively. Section 5 derives analytical results on their comparative efficiency, which are supplemented by numerical calculations.

## 2 The model

We study a team, that is, a group of agents with a common objective and to each of whom is assigned a specific set of choice variables relevant to the team's payoff. Although centralizing these choices might be desirable on informational grounds, we regard it as technologically infeasible. Thus, the problem we study is a by-product of the dispersions of actions and of information. More specifically, it is due to the fact that some agents have information relevant to other agents' decisions.

We consider a two-member organization that must take four decisions. Agent 1 controls  $x_1$  and  $y_1$ , and agent 2 controls  $x_2$  and  $y_2$ . The organization operates in an uncertain environment with two unknown parameters  $a$  and  $b$ . Information is dispersed. Each agent observes both  $a$  and  $b$  with error. Specifically, agent  $i$  sees  $\alpha_i$  and  $\beta_i$ , where

$$\begin{aligned}\alpha_i &= a + \epsilon_i, \\ \beta_i &= b + \eta_i.\end{aligned}$$

We assume that  $a$ ,  $b$ ,  $\epsilon_1$ ,  $\epsilon_2$ ,  $\eta_1$ , and  $\eta_2$  are all Gaussian mean zero independently distributed random variables.

The organization's goal is to minimize the expectation of the loss function

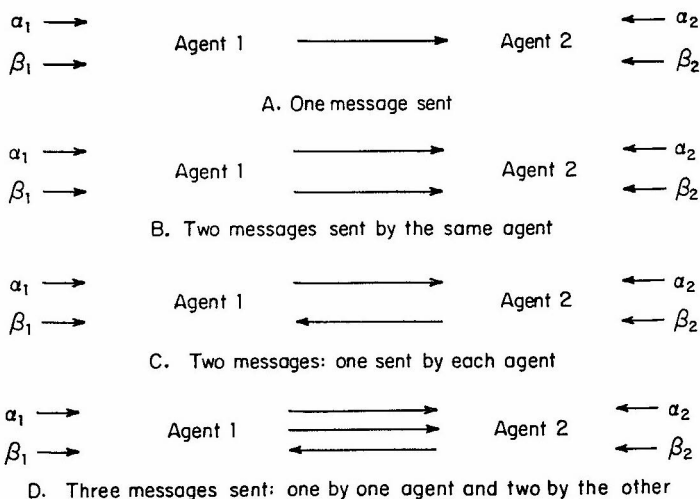
$$L = (x_1 + x_2 - a)^2 + (y_1 + y_2 - b)^2 + \lambda(x_1 - x_2)^2 + \mu(y_1 - y_2)^2, \quad (1)$$

where  $\lambda, \mu \geq 0$  are fixed constants. The first two terms express the objective of taking collective actions that approximate the targets  $a$  and  $b$ . The third and fourth terms represent the costs incurred due to the imperfectly coordinated individual actions.

We take the decentralization of actions across team members as immutable. The problem studied in this chapter is the design of the communication system that agents use to share their information. The actions of the agents can be based on the observations they receive directly and on whatever additional information they receive through the chosen communication system.

We are interested in comparing the efficiency of forms of communication that use the same number of messages transmitted. To share all information completely would require four messages to be sent – each would have to send both observations to the other. This would achieve the first-best results.

The possible *forms* of communication for fewer than four messages can be represented in the following diagrams:



The arrows in these diagrams represent a single real number transmitted in the indicated direction. Notice that in every case there is another

communication structure in which the direction of the transmission is reversed.

In this simple model, it is only in the case of exactly two messages being sent that there is a *real choice of structural form* in the pattern of communication – B versus C. B models a centralized organization while C describes a more symmetric type of interchange of information between the players.<sup>2</sup> Consequently, the focus of the formal analysis below is limited to these two cases.

The choice of communication structure is based on the team's a priori beliefs about its environment as well as on the information available to its members. Using the principle of person-by-person optimality (Marschak and Radner 1972, Chapter 5, Section 3), the best team decision rule involves each agent computing his posterior beliefs based on all the information available, and then optimizing the team's objective function. We concentrate on the case in which the available information greatly reduces the a priori uncertainty of each agent about the parameters  $a$  and  $b$ . We therefore derive asymptotic results that apply as the variances of the prior distributions of  $a$  and  $b$ , respectively  $\sigma_a^2$  and  $\sigma_b^2$ , become very large. These results are much sharper than what would be obtained in the general case of an optimal Bayesian decision rule with an informative prior.

### 3 Interchange of information

The form of communication in diagram C will be termed *interchange*, as it involves having the agents send messages to each other. Each agent then controls two variables based on three inputs, the two observations seen personally and the message received from the other. We restrict attention to transmissions that are linear combinations of the observations each player receives. Actually, without loss of generality we can consider convex combinations, which are denoted

$$z_1 = c\alpha_1 + (1-c)\beta_1, \quad z_2 = d\alpha_2 + (1-d)\beta_2.$$

The decisions of agent 1 are

$$x_1 = ez_2 + f\alpha_2 + g\beta_1, \quad y_1 = hz_2 + k\alpha_1 + \ell\beta_1.$$

Note, however, that  $g$  and  $k$  must both be zero. Information about  $b$  contained in  $\beta_1$  is completely irrelevant to the choice of  $x_1$  since  $a$  and  $b$  are independent. Likewise  $k=0$  would make  $y_1$  dependent on  $a$ , and hence would introduce noise in the objective function. Similar considerations apply to agent 2.

Therefore we let the decisions be given by

$$\begin{aligned}x_1 &= ez_2 + f\alpha_1, & y_1 &= hz_2 + \ell\beta_1, \\x_2 &= mz_1 + n\alpha_2, & y_2 &= qz_1 + s\beta_2.\end{aligned}$$

Substituting the communication and decision rules into (1), we find that the coefficients of  $\sigma_a^2$  and  $\sigma_b^2$  are, respectively,

$$(ed + f + mc + n - 1)^2 + (hd + qc)^2 + \lambda(ed + f - mc - n)^2 + \mu(hd - qc)^2, \quad (2)$$

and

$$\begin{aligned}[e(1-d) + m(1-c)]^2 + [h(1-d) + \ell + q(1-c) + s - 1]^2 \\ + \lambda[e(1-d) - m(1-c)]^2 + \mu[h(1-d) + \ell - q(1-c) - s]^2.\end{aligned} \quad (3)$$

Setting each of the eight squared expressions above to zero<sup>3</sup> gives the relationships

$$\begin{aligned}ed + f &= \frac{1}{2}, & mc + n &= \frac{1}{2}, \\hd &= 0, & qc &= 0, \\e(1-d) &= 0, & m(1-c) &= 0, \\h(1-d) + \ell &= \frac{1}{2}, & q(1-c) + s &= \frac{1}{2}.\end{aligned} \quad (4)$$

The simplest way to solve the overall minimization problem is to consider  $c$  and  $d$  as choice variables and to let the other eight variables be determined by the above. Because of the multiplicative form of the restrictions (4), we have several cases to consider. The cases are divided according to whether  $c$  and  $d$  are zero, one, or something in between. This would produce nine cases in all. However, only the four in which  $c$  and  $d$  have an extreme value will be of interest.

To see why the other five cases can be eliminated, consider for example the restrictions implied by equation (4) when  $c$  and  $d$  are both between zero and one. Here we have directly that

$$h = q = e = m = 0, \quad f = n = \ell = s = \frac{1}{2}.$$

The meaning of these restrictions is that any information received from the other agent is ignored in the decision rule. The agents behave in an entirely decentralized fashion, setting  $x_i$  and  $y_i$  at one-half of the mean of the distributions of  $a$  and  $b$ , respectively.

In general, whenever one agent chooses to transmit a mixed signal, the restrictions of equation (4) imply that the transmission is not used by the

other agent. Intuitively, a mixed signal cannot be interpreted by the receiver who cannot decompose the signal into a piece relevant for action  $x$  and a piece relevant for action  $y$ .

Thus, we concentrate on the four remaining cases:

**Case 1.**  $c=0$ ,  $d=0$ ; interchange of information on  $b$  implying the following constraints:

$$\begin{array}{l}
 \left. \begin{array}{l} e=0 \\ f=\frac{1}{2} \end{array} \right\} \begin{array}{l} \text{agent 1 uses only private information} \\ \text{on } a \text{ to make } x \text{ decision} \end{array} \\
 \left. \begin{array}{l} m=0 \\ n=\frac{1}{2} \end{array} \right\} \begin{array}{l} \text{agent 2 uses only private information} \\ \text{on } a \text{ to make } x \text{ decision} \end{array} \\
 \left. \begin{array}{l} h+l=\frac{1}{2} \\ q+s=\frac{1}{2} \end{array} \right\} \begin{array}{l} \text{weights of own and exchanged information} \\ \text{to make } y \text{ decision must sum to } \frac{1}{2} \end{array}
 \end{array} \quad (5)$$

**Case 2.**  $c=0$ ,  $d=1$ ; agent 1 transmits information on  $b$ , agent 2 transmits information on  $a$ , implying the following constraints:

$$\begin{array}{l}
 e+f=\frac{1}{2} \quad \text{agent 1 uses both information to set } x_1 \\
 \left. \begin{array}{l} m=0 \\ n=\frac{1}{2} \end{array} \right\} \begin{array}{l} \text{agent 2 uses only private information} \\ \text{on } a \text{ to make } x \text{ decision} \end{array} \\
 \left. \begin{array}{l} h=0 \\ \ell=\frac{1}{2} \end{array} \right\} \begin{array}{l} \text{agent 1 uses only private information} \\ \text{on } b \text{ to make } y \text{ decision} \end{array} \\
 q+x=\frac{1}{2} \quad \text{agent 2 uses both information to set } y_2
 \end{array} \quad (6)$$

**Case 3.**  $c=1$ ,  $d=0$ ; agent 1 transmits information on  $a$ , agent 2 transmits information on  $b$ , implying the following constraints:

$$\begin{array}{l}
 e=0, \quad f=\frac{1}{2}, \\
 h+l=\frac{1}{2}, \quad m+n=\frac{1}{2}, \\
 q=0, \quad s=\frac{1}{2}.
 \end{array} \quad (7)$$

**Case 4.**  $c=1$ ,  $d=1$ ; interchange of information on  $a$ , implying the following constraints:

$$\begin{array}{l}
 e+f=\frac{1}{2}, \quad h=0, \\
 \ell=\frac{1}{2}, \quad m+n=\frac{1}{2}, \\
 q=0, \quad s=\frac{1}{2}.
 \end{array} \quad (8)$$

Now, to minimize the expectation of the loss function in each case, we substitute the conditions derived above into equation (1) and minimize the remaining expression. Having set equations (2) and (3) to zero because we consider the asymptotic case of large prior uncertainty, the remaining expression in the minimand involves only the four error variances  $\sigma_{\epsilon_1}^2$ ,  $\sigma_{\epsilon_2}^2$ ,  $\sigma_{\eta_1}^2$  and  $\sigma_{\eta_2}^2$ .

It turns out that in each case the objective function is convex. For example, in case 1, the optimum is given by

$$q = \ell = \frac{1}{2} \frac{\sigma_{\eta_2}^2}{\sigma_{\eta_1}^2 + \sigma_{\eta_2}^2} \quad (9)$$

with the other parameters determined directly from equation (4). The value of the loss function at this optimum is

$$L_{11} = \frac{1}{4} (1 + \lambda) (\sigma_{\epsilon_1}^2 + \sigma_{\epsilon_2}^2) + \frac{\sigma_{\eta_1}^2 \sigma_{\eta_2}^2}{\sigma_{\eta_1}^2 + \sigma_{\eta_2}^2}. \quad (10)$$

Before considering the other cases, the economic interpretation of these conditions should be discussed.

As information about  $b$  has been transmitted, the full state of knowledge about  $b$  is perceived in common by the two agents. The decision rule is set up to produce  $y_1 = y_2$  at the value of one-half the conditional mean of  $b$  given both observations. Indeed, this eliminates the cost of malcoordination because taking the best action is conditional on all the information about  $b$ . As no information about  $a$  is available, other than the direct observation of each agent, the organization is at risk with respect to errors both of estimation and of malcoordination. Each agent's own error of estimation contributes separately to the former, as there is no reduction due to pooling of observations. Each agent bases his or her action  $x$  on personal information about  $a$ , neglecting the cost of malcoordination. This occurs because  $\sigma_a^2$  is very large. An attempt to coordinate should be based on the prior information, which is too negative for this course of action to be worth it.

Case 2 is somewhat different in structure. The parameters should be chosen at

$$f = \frac{\sigma_{\epsilon_2}^2}{\sigma_{\epsilon_1}^2 + \sigma_{\epsilon_2}^2} \left( \frac{1}{1 + \lambda} \right), \quad s = \frac{\sigma_{\eta_1}^2}{\sigma_{\eta_1}^2 + \sigma_{\eta_2}^2} \left( \frac{1}{1 + \mu} \right). \quad (11)$$

The value of the loss function is

$$L_{12} = \sigma_{\epsilon_2}^2 \left( 1 - \frac{\sigma_{\epsilon_2}^2}{(1 + \lambda)(\sigma_{\epsilon_1}^2 + \sigma_{\epsilon_2}^2)} \right) + \sigma_{\eta_1}^2 \left( 1 - \frac{\sigma_{\eta_1}^2}{(1 + \mu)(\sigma_{\eta_1}^2 + \sigma_{\eta_2}^2)} \right). \quad (12)$$



In this case, agent 1's information about  $b$  is transmitted to agent 2 and agent 2's information about  $a$  is transmitted to agent 1. Therefore agent 1 is in a good position to control  $x$ , having the better information *ex post*. On the other hand, to the extent that agent 2 does not choose the same level of  $x_2$  as agent 1 chooses for  $x_1$ , they are at risk for the coordination loss, and increasingly so as  $\lambda$  becomes large. Equation (11) shows that as  $\lambda$  becomes large agent 2 uses less and less of his or her own information to avoid malcoordination.

These considerations can be useful in deriving an upper bound on the loss in this mode. This loss is achieved by using a decision rule that we call the *mimic strategy*, which eliminates the risk due to malcoordination of the  $x$  and  $y$  decisions.

With respect to the information relevant to the choice of  $x_1$  and  $x_2$ , as the decision rules are common knowledge, the agent can choose  $x_1 = x_2$  identically. For finite  $\lambda$  this will not be optimal, but as  $\lambda \rightarrow \infty$ , the agent ignores his or her own information and sets  $x_1 = \frac{1}{2}x_2 = \frac{1}{2}\beta_2$ .

Precisely the same considerations apply, of course, for player 2 and the  $y$  decisions.

As this "mimic" strategy is always available, the worst that can happen under case 2 is to use player 2's information about  $a$  to choose  $x_1 = x_2$  and player 1's information about  $b$  to choose  $y_1 = y_2$ , while ignoring 2's information about  $b$  and 1's information about  $a$ . Algebraically, the loss  $L_{12}$  is at most

$$\sigma_{\eta_1}^2 + \sigma_{\epsilon_2}^2. \quad (13)$$

Case 3 parallels case 2. The loss is

$$L_{13} = \sigma_{\epsilon_1}^2 \left( 1 + \frac{\sigma_{\epsilon_1}^2}{(1+\lambda)(\sigma_{\epsilon_1}^2 + \sigma_{\epsilon_2}^2)} \right) + \sigma_{\eta_2}^2 \left( 1 - \frac{\sigma_{\eta_2}^2}{(1+\mu)(\sigma_{\eta_1}^2 + \sigma_{\eta_2}^2)} \right). \quad (14)$$

Case 4 parallels case 1; the loss is

$$L_{14} = \frac{1}{4}(1+\mu)(\sigma_{\eta_1}^2 + \sigma_{\eta_2}^2) + \frac{\sigma_{\epsilon_1}^2 \sigma_{\epsilon_2}^2}{\sigma_{\epsilon_1}^2 + \sigma_{\epsilon_2}^2}. \quad (15)$$

For any combination of parameters, the optimal use of the interchange mode will be determined by which of the four cases achieves the minimum loss.

If  $\lambda$  (or  $\mu$ ) equals zero, the above method does not apply. It is easy to see that the first best is then achievable by the following strategy. Since  $\lambda = 0$ , there is no cost in malcoordination of the actions  $x_1$  and  $x_2$  and the optimal predictor of  $a$  is

$$\frac{\sigma_{\epsilon_2}^2}{\sigma_{\epsilon_1}^2 + \sigma_{\epsilon_2}^2} \alpha_1 + \frac{\sigma_{\epsilon_1}^2}{\sigma_{\epsilon_1}^2 + \sigma_{\epsilon_2}^2} \alpha_2, \quad (16)$$

which is linear in  $\alpha_1$  and  $\alpha_2$  and can be reached in a decentralized way at no cost. That is, agent  $i$  sets  $x_i$  equal to the corresponding term of equation (16), that is,

$$x_1 = \frac{\sigma_{\epsilon_2}^2}{\sigma_{\epsilon_1}^2 + \sigma_{\epsilon_2}^2} \alpha_1 \quad \text{and} \quad x_2 = \frac{\sigma_{\epsilon_1}^2}{\sigma_{\epsilon_1}^2 + \sigma_{\epsilon_2}^2} \alpha_2.$$

The two channels of communication can then be used to pool all available information about  $b$ . Thus, as all information is then common knowledge, each agent can choose one-half of the optimal predictor, causing no loss of coordination. The expected loss is

$$L = \frac{\sigma_{\epsilon_1}^2 \sigma_{\epsilon_2}^2}{\sigma_{\epsilon_1}^2 + \sigma_{\epsilon_2}^2} + \frac{\sigma_{\eta_1}^2 \sigma_{\eta_2}^2}{\sigma_{\eta_1}^2 + \sigma_{\eta_2}^2}.$$

When  $\lambda \neq 0$ ,  $\mu \neq 0$ , as seen above, the cost of malcoordination forces the team to use the predictor

$$\frac{1}{2}(\alpha_1 + \alpha_2). \quad (17)$$

This is because we are considering the asymptotic case where  $\sigma_a^2$  goes to infinity. If  $\sigma_a^2$  is finite, the optimal action would be a combination of equations (16) and (17).

#### 4 Centralization of information

We consider centralization of information in the hands of agents 1 and 2, which we refer to as cases 1 and 2 of the centralization mode. In case 1, where agent 2 sends two messages to agent 1, he or she will obviously send two observations,

$$\alpha_2 = a + \epsilon_2, \quad \beta_2 = b + \eta_2.$$

Using the principle of person-to-person optimality in Marschak and Radner [(1972) Chapter 5, Section 3] we can set

$$x_2 = \frac{1}{2}\alpha_2, \quad y_2 = \frac{1}{2}\beta_2. \quad (18)$$

Substituting equation (18) in the loss function of the team we have:

$$L = [x_1 + \frac{1}{2}(\epsilon_2 - a)]^2 + [y_1 + \frac{1}{2}(\eta_2 - b)]^2 \\ + \lambda [x_1 - \frac{1}{2}(a + \epsilon_2)]^2 + \mu [y_1 - \frac{1}{2}(b + \eta_2)]^2.$$

In view of the independence of the errors, the decision rule of agent 1 is

of the form

$$x_1 = e\alpha_1 + f\alpha_2, \quad y_1 = g\beta_1 + h\beta_2. \quad (19)$$

Substituting equation (19) in the loss function and maximizing the expected value of the loss given the information  $(\alpha_1, \alpha_2, \beta_1, \beta_2)$  yields

$$e = \frac{\sigma_{\epsilon_2}^2}{(1+\lambda)(\sigma_{\epsilon_1}^2 + \sigma_{\epsilon_2}^2)}, \quad g = \frac{\sigma_{\eta_2}^2}{(1+\mu)(\sigma_{\eta_1}^2 + \sigma_{\eta_2}^2)},$$

$$f = \frac{1}{2} - e, \quad h = \frac{1}{2} - g.$$

The value of the expected loss in case 1 is

$$L_{C1} = \sigma_{\epsilon_2}^2 \left( 1 - \frac{\sigma_{\epsilon_2}^2}{(1+\lambda)(\sigma_{\epsilon_1}^2 + \sigma_{\epsilon_2}^2)} \right) + \sigma_{\eta_2}^2 \left( 1 - \frac{\sigma_{\eta_2}^2}{(1+\mu)(\sigma_{\eta_1}^2 + \sigma_{\eta_2}^2)} \right). \quad (20)$$

Symmetrically (case 2) if agent 1 transmits both observations to agent 2, we find an optimal expected value of the loss

$$L_{C2} = \sigma_{\epsilon_1}^2 \left( 1 - \frac{\sigma_{\epsilon_1}^2}{(1+\lambda)(\sigma_{\epsilon_1}^2 + \sigma_{\epsilon_2}^2)} \right) + \sigma_{\eta_1}^2 \left( 1 - \frac{\sigma_{\eta_1}^2}{(1+\mu)(\sigma_{\eta_1}^2 + \sigma_{\eta_2}^2)} \right). \quad (21)$$

## 5 Comparison of communication forms

When  $\lambda$  and  $\mu$  are very large, cases 1 and 4 of interchange become irrelevant because they provide no way of avoiding the costs of malcoordination. The reason is that they specialize exchange of information on either  $b$  (or  $a$ ), obliging the agents to take the  $x$  decision ( $y$  decision) on the basis of their diverse private information. Since  $\sigma_a^2$  is large, they cannot rely on a common prior.

As  $\lambda$  and  $\mu$  go to infinity, the asymptotic loss in the centralization mode is

$$\inf(\sigma_{\epsilon_1}^2 + \sigma_{\eta_1}^2, \sigma_{\epsilon_2}^2 + \sigma_{\eta_2}^2).$$

For example, if  $\sigma_{\epsilon_1}^2 + \sigma_{\eta_1}^2 < \sigma_{\epsilon_2}^2 + \sigma_{\eta_2}^2$ , that is, agent 1 is a better observer, there must be centralization in the hands of agent 2. Then, both agents can take the same decisions (to avoid the costs of malcoordination) based on the best common information structure that is possible given the informational constraints.

Similarly, in the interchange mode the asymptotic loss is

$$\inf(\sigma_{\epsilon_1}^2 + \sigma_{\eta_2}^2, \sigma_{\epsilon_2}^2 + \sigma_{\eta_1}^2).$$

The best mode depends on what is the best combination of good information in terms of the sum of the variances.

The optimal form of communication is then easily derived from the best observers. If one agent is the best observer of both parameters (e.g.,  $\sigma_{\epsilon_1}^2 < \sigma_{\epsilon_2}^2$ ,  $\sigma_{\eta_1}^2 < \sigma_{\eta_2}^2$ ) we have centralization (in the hands of the bad observer). If each agent is the best observer in one parameter, we have interchange, in which for each parameter the transmission is made by the good observer. *The most accurate observer of each parameter should transmit his observation to the other agent.*

On the other extreme, when  $\lambda = 0$  (or  $\mu = 0$ ) the first best is achieved by the interchange mode case 1 (or case 4). The optimal predictor of  $a$  (respectively,  $b$ ) is achieved in a decentralized way without worrying about malcoordination of  $x$  (respectively,  $y$ ) and full exchange of information is possible on  $b$  (respectively,  $a$ ) so that the optimal predictor of  $b$  (respectively,  $a$ ) is also achieved but here with  $y_1 = y_2$  (respectively,  $x_1 = x_2$ ) which avoids any cost of malcoordination.

In this case the centralization mode cannot reach the first best. For example, if agent 1 transmits information, action  $y_1$  will have to be based only on partial information and a loss will be incurred. The best form of centralization depends on the relative values of  $\mu$ ,  $\sigma_{\epsilon_1}^2$ ,  $\sigma_{\epsilon_2}^2$ ,  $\sigma_{\eta_1}^2$ , and  $\sigma_{\eta_2}^2$ , in the case  $\lambda = 0$  as seen in equations (20) and (21). Table 1 illustrates the domination of the interchange mode 1 (because  $\lambda = 0$ ) and gives in each case the best form of centralization.

When  $\lambda$  and  $\mu$  are finite, we are in a case intermediary between the two extremes described above and the comparison is more subtle, as seen in Tables 2 and 3. A necessary condition for centralization to win is that the information of an agent be better than the information of the other agent concerning both variables. But this is not sufficient. This dominance ensures the superiority of centralization to cases 2 and 3 of the interchange mode but not necessarily to cases 1 and 4, if  $\lambda$  and  $\mu$  are small enough.

A typical entry in the tables is as follows:

centralization loss	optimal case
interchange loss	optimal case

Table 1.  $\sigma_{\epsilon_1}^2 = 1$ ,  $\lambda = 0 = \mu = 10$ . $\sigma_{\epsilon_2}^2 = 1$ 

		$\sigma_{\eta_1}^2 \backslash \sigma_{\eta_2}^2$		
		1	5	9
1	2	1.46	1.49	1.49
	1	1.00	1.33	1.40
5	2	1.49	5.27	5.34
	1	1.33	3.00	3.71
9	2	1.49	5.34	9.09
	1	1.40	3.71	5.00

 $\sigma_{\epsilon_2}^2 = 5$ 

		$\sigma_{\eta_1}^2 \backslash \sigma_{\eta_2}^2$		
		1	5	9
1	1	1.75	1.82	1.82
	1	1.33	1.67	1.73
5	2	1.82	5.61	5.67
	1	1.67	3.33	4.05
9	2	1.82	5.67	9.42
	1	1.73	4.05	5.33

 $\sigma_{\epsilon_2}^2 = 9$ 

		$\sigma_{\eta_1}^2 \backslash \sigma_{\eta_2}^2$		
		1	5	9
1	2	1.86	1.89	1.89
	1	1.40	1.73	1.80
5	2	1.89	5.67	5.74
	1	1.73	3.40	4.11
9	2	1.89	5.74	9.49
	1	1.80	4.11	5.40

Table 2.  $\sigma_{\epsilon_1}^2 = 1$ ,  $\lambda = \mu = 2$ .

$\sigma_{\epsilon_2}^2 = 1$

	$\sigma_{\eta_1}^2 \backslash \sigma_{\eta_2}^2$		
	1	5	9
1	1.67 1	1.78 2	1.80 3
	1.67 2	1.78 2	1.80 2
5	1.78 1	5.00 1	5.24 2
	1.78 3	4.00 1	4.71 1
9	1.80 1	5.24 2	8.33 1
	1.80 3	4.71 1	6.00 1

$\sigma_{\epsilon_2}^2 = 5$

	$\sigma_{\eta_1}^2 \backslash \sigma_{\eta_2}^2$		
	1	5	9
1	1.78 2	1.89 2	1.91 2
	1.78 3	4.56 3	4.58 2
5	4.56 1	5.11 2	5.35 2
	1.89 3	5.11 1	7.71 1
9	4.58 1	8.02 1	8.44 2
	1.91 3	5.35 3	8.44 3

$\sigma_{\epsilon_2}^2 = 9$

	$\sigma_{\eta_1}^2 \backslash \sigma_{\eta_2}^2$		
	1	5	9
1	1.80 2	1.91 2	1.93 2
	1.80 3	4.58 3	7.27 3
5	4.58 2	5.13 2	5.37 2
	1.91 3	5.13 3	8.04 3
9	7.27 2	8.04 2	8.47 2
	1.93 3	5.37 3	8.47 3

Table 3.  $\sigma_{\epsilon_1}^2 = 1$ ,  $\lambda = 1 = \mu = 10$ . $\sigma_{\epsilon_2}^2 = 1$ 

		$\sigma_{\eta_1}^2 \backslash \sigma_{\eta_2}^2$		
		1	5	9
1	1	1.71 <sub>1</sub>	1.74 <sub>2</sub>	1.74 <sub>2</sub>
	5	1.50 <sub>1</sub>	1.74 <sub>2</sub>	1.74 <sub>2</sub>
5	1	1.74 <sub>1</sub>	5.52 <sub>1</sub>	5.59 <sub>2</sub>
	5	1.74 <sub>3</sub>	3.50 <sub>1</sub>	4.21 <sub>1</sub>
9	1	1.74 <sub>1</sub>	5.59 <sub>1</sub>	9.34 <sub>1</sub>
	5	1.74 <sub>3</sub>	4.21 <sub>1</sub>	5.50 <sub>1</sub>

 $\sigma_{\epsilon_2}^2 = 5$ 

		$\sigma_{\eta_1}^2 \backslash \sigma_{\eta_2}^2$		
		1	5	9
1	1	1.87 <sub>2</sub>	1.90 <sub>2</sub>	1.90 <sub>2</sub>
	5	1.87 <sub>3</sub>	3.83 <sub>1</sub>	3.90 <sub>1</sub>
5	1	3.90 <sub>1</sub>	5.69 <sub>2</sub>	5.75 <sub>2</sub>
	5	1.90 <sub>3</sub>	5.50 <sub>1</sub>	6.21 <sub>1</sub>
9	1	3.91 <sub>1</sub>	7.75 <sub>1</sub>	9.51 <sub>2</sub>
	5	1.90 <sub>3</sub>	5.75 <sub>3</sub>	7.50 <sub>1</sub>

 $\sigma_{\epsilon_2}^2 = 9$ 

		$\sigma_{\eta_1}^2 \backslash \sigma_{\eta_2}^2$		
		1	5	9
1	1	1.91 <sub>2</sub>	1.94 <sub>2</sub>	1.94 <sub>2</sub>
	5	1.91 <sub>3</sub>	5.57 <sub>3</sub>	5.90 <sub>1</sub>
5	1	5.57 <sub>2</sub>	5.72 <sub>2</sub>	5.79 <sub>2</sub>
	5	1.93 <sub>3</sub>	5.72 <sub>3</sub>	8.21 <sub>1</sub>
9	1	5.94 <sub>1</sub>	9.42 <sub>2</sub>	9.54 <sub>2</sub>
	5	1.94 <sub>3</sub>	5.79 <sub>3</sub>	9.50 <sub>1</sub>

## NOTES

- 1 The earlier literature includes theoretical papers by Hurwicz (1951), Beckman (1953), Marschak (1954) as well as experimental papers by Bavelas (1951). See also the more recent work of Miyasawa (1967), Oniki (1974) and Groves and Radner (1972).
- 2 As stressed in the introduction, implicitly, we assume that the costs of coding and decoding are identical and without increasing returns. In mode B, agent 1 encodes two messages and agent 2 decodes two messages. In mode C, each agent encodes one message and decodes one message.
- 3 We are assuming here that  $\lambda \neq 0$ ,  $\mu \neq 0$ . See below the treatment of the cases where  $\lambda = 0$  or  $\mu = 0$ .

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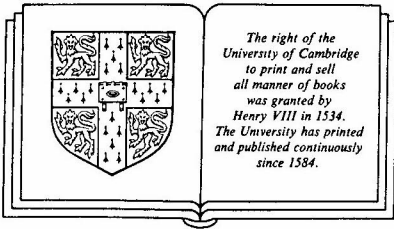
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